$\Rightarrow 0 < u_{n+1} \leqslant 1$

Conclusion: $\forall n \in \mathbb{N};\ 0 < u_n \leqslant 1$

2°) $\forall n \in \mathbb{N}$;

$$u_{n+1} - \frac{1}{2}u_n = f(u_n) - \frac{1}{2}u_n$$

$$= \frac{\sqrt{1+u_n^2} - 1}{u_n} - \frac{1}{2}u_n$$

$$= \frac{2\sqrt{1+u_n^2} - 2 - u_n^2}{2u_n}$$

$$= \frac{2\sqrt{1+u_n^2} - (2+u_n^2)}{2u_n} \times \frac{2\sqrt{1+u_n^2} + (2+u_n^2)}{2\sqrt{1+u_n^2} + (2+u_n^2)}$$

$$= \frac{4(1+u_n^2) - (2+u_n^2)^2}{2u_n(2\sqrt{1+u_n^2} + 2 + u_n^2)}$$

$$= \frac{4 + 4u_n^2 - 4 - 4u_n^2 - u_n^4}{2u_n(2\sqrt{1+u_n^2} + 2 + u_n^2)}$$

$$= \frac{-u_n^4}{2u_n(2\sqrt{1+u_n^2} + 2 + u_n^2)} \leqslant 0$$

Donc $u_{n+1} \leqslant \frac{1}{2}u_n \quad \forall n \in \mathbb{N}$

Qarray.tn-Aqra Blech- Qarray.tn